[This question paper contains 6 printed pages.]

Sr. No. of Question Paper : 1796 **GC-3** **Your Roll No...............**

Unique Paper Code : 32371101

Name of the Paper : Descriptive Statistics

Name of the Course : **B.Sc. (Hons.) Statistics under CBCS**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **6** questions in all.
3. Question No. **1** is compulsory.
4. Attempt 5 more questions selecting **three** questions from **Section A** and **two** from **Section B.**
5. Use of simple calculator is allowed.

1. Fill in the blanks :

   (i) $|x + 6| + |x - 4| + |x| + |x + 10| + |x + 3|$ is least for x = ______ .

   (ii) For a platykurtic distribution γ_2 is ______ .

   (iii) For a discrete distribution standard deviation is ______ than mean deviation about mean.

   (iv) If Corr(X, Y) = 0.8, $\sigma_x = 2.5$ and $\sigma_y = 3.5$, then Var(3X-2Y) is ______ .

   (v) If X_1, X_2, and X_3 are three variables, then partial correlation coefficient $r_{23.1}$ = ______ .

P.T.O.

(vi) Correlation coefficient is the ______ of regression coefficients.

(vii) The acute angle between two lines of regression is given by ______ .

(viii) In case of n attributes, the total number of ultimate class frequencies is ______ and number of positive class frequencies is ______ .

(ix) If $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then lower limit of $P(A \cap B)$ is ______ .

(x) Milk is sold at the rates of 8, 10 and 12 rupees per litre in three different months. Assuming that equal amounts were spent on milk by a family in the three months, the average price of milk is ______ .

(xi) Arithmetic mean of 100 observations is 50 and standard deviation is 10. If 5 is subtracted from each of the observations and then it is divided by 4 then new arithmetic mean is ______ and standard deviation is ______ .

(xii) A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. If P(B) = (3/2), P(A) and P(C) = (½) P(B) then P(A) is ______ and $P(\bar{A} \cap \bar{B})$ is ______ . (1,1,1,1,1,1,1,1,2,2,2)

SECTION A

2. (a) (i) Prove that the sum of the squares of the deviations of a set of observations is minimum when taken about mean.

(ii) Let r be the range and s be the standard deviation of a set of observations $x_1, x_2, ..., x_n$. Prove that $s \leq r$.

(b) In a frequency table, the upper boundary of each class interval has a constant ratio to the lower boundary. Show that the geometric mean G may be expressed by the following formula :

$$\log G = x_0 + \frac{c}{N}\sum_i f_i (i-1),$$

where, x_0 is the logarithm of the mid value of the first interval and c is the logarithm of the ratio between upper and lower boundaries. (6,6)

3. (a) Show that in a discrete series if deviations $x_i = X_i - M$, are small compared with the value of the mean M so that $(x/M)^3$ and higher powers of (x/M) are neglected,

(i) $H = M\left(1 - \frac{\sigma^2}{M^2}\right)$

(ii) Mean $\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{M}\left(1 + \frac{3\sigma^2}{8M^2}\right)$ approx.

where, H is the harmonic mean of the values $x_1, x_2, \ldots, x_n$ and σ^2 is the variance.

(b) Two variables X and Y are known to be related to each other by the relation $Y = \frac{X}{aX+b}$. Derive the normal equations for fitting the given curve and estimate the constants 'a' and 'b' for a given set of n points $\{(x_i, y_i), i = 1, 2 \ldots\ldots n\}$. (7,5)

4. (a) Define Spearman's rank correlation coefficient. Let $x_1, x_2, ..., x_n$ be the ranks of n individuals according to a character A and $y_1, y_2, ..., y_n$ be the corresponding ranks of the individuals according to another character B. Obtain the rank correlation coefficient between them if $x_i + y_i = n + 1 \; \forall \; i = 1, 2,....n$.

(b) X and Y are two random variables with variances σ_x^2 and σ_y^2 respectively and r is the coefficient of correlation between them. If $U = X + kY$ and

$V = X + \frac{\sigma_x}{\sigma_y} Y$, find the value of k so that U and V are uncorrelated.

(6,6)

5. (a) Show that $1 - R_{1.23}^2 = \left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)$

Deduce that (ii) $R_{1.23} \geq r_{12}$

(iii) $R_{1.23}^2 = r_{12}^2 + r_{13}^2$, if $r_{23} = 0$

(iv) $1 - R_{1.23}^2 = \frac{(1-\rho)(1+2\rho)}{(1+\rho)}$, provided all coefficients of zero order are equal to ρ.

(b) Given that $Y = kX + 4$ and $X = 4Y + 5$ are the lines of regression of Y on X and X on Y respectively, show that $0 \leq k \leq 1/4$. If $k = \frac{1}{16}$, find mean of two variables and the coefficient of correlation between them.

(7,5)

SECTION B

6\. (a) Four tickets marked 00, 01, 10 and 11 respectively are placed in a bag. A ticket is drawn at random five times, being replaced each time. Find the probability that the sum of the numbers on the tickets thus drawn is 23.

(b) If $A_1, A_2, ..., A_n$ are n independent events with $P(A_i) = 1 - \frac{1}{\alpha^i}$, $i = 1, 2, ..., n$ then find the value of $P(A_1 \cup A_2 \cup ... \cup A_n)$.

(c) A problem in Statistics is given to three students A, B and C, whose chance of solving it are ½, ¾ and ¼ respectively. What is the probability that the problem will be solved if all of them try independently. (5,3,4)

7\. (a) State Bayes' theorem.

In answering a multiple choice test, an examinee either knows the answer or he guesses or he copies. Suppose each question has four choices. Let the probability that examinee copies the answer is 1/6 and the probability that he guesses is 1/3. The probability that his answer is correct given that he copied the answer is 1/8. Suppose an examinee answers a question correctly, what is the probability that he really knows the answer ?

(b) If $\frac{(A)}{N} = x$, $\frac{(B)}{N} = 2x$, $\frac{(C)}{N} = 3x$ and $\frac{(AB)}{N} = \frac{(BC)}{N} = \frac{(CA)}{N} = y$,

then, using the conditions of consistency of attributes show that

$$0 < y \leq x \leq \frac{1}{4}.$$

(7,5)

8. (a) Let $A_1, A_2, \ldots, A_n$ be the events in the domain of probability function P, such that $P\left[\bigcup_{i=1}^{n} A_i\right] \le \sum_{i=1}^{n} P[A_i]$. Using this relationship, prove that :

(i) $P\left[\bigcap_{i=1}^{n} A_i\right] \ge 1 - \sum_{i=1}^{n} P\left[\bar{A}_i\right]$, and

(ii) $P\left[\bigcap_{i=1}^{n} A_i\right] \ge \sum_{i=1}^{n} P[A_i] - (n-1)$.

(b) Given that (A) = (α) = (B) = (β) = (C) = (γ) = N/2 and (ABC) = (αβγ), then show that

$$2(ABC) = (AB) + (AC) + (BC) - N/2.$$ (7,5)